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Singlet Deformation and Non(anti)commutative $\mathcal{N} = 2$ Supersymmetric $U(1)$ Gauge Theory

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Abstract

We study $\mathcal{N} = 2$ supersymmetric $U(1)$ gauge theory in non(anti)commutative $\mathcal{N} = 2$ harmonic superspace with the singlet deformation, which preserves chirality. We construct a Lagrangian which is invariant under both the deformed gauge and supersymmetry transformations. We find the field redefinition such that the $\mathcal{N} = 2$ vector multiplet transforms canonically under the deformed symmetries.

Non(anti)commutative superspace[1] with nonanticommutativity in Grassmann odd coordinates appears in superstrings compactified on Calabi-Yau threefold in the graviphoton background [2, 3, 4]. The low-energy effective theory on the D-brane is realized by supersymmetric gauge theories in non(anti)commutative superspace. Perturbative and non-perturbative aspects of these gauge theories have been studied extensively[5, 6, 7, 8]. It is an interesting problem to study the deformation of extended superspace since it admits a variety of deformation parameters[9]. The deformation of extended superspace has been recently studied in [10, 11, 12, 13, 14, 15]. In a previous paper [12], we have studied the deformed Lagrangian explicitly in the component formalism up to the first order in the deformation parameter C . Since the Lagrangian get higher order correction in C , the full Lagrangian is rather complicated. Moreover, the harmonic superspace formalism introduces the infinite number of auxiliary fields. In order to preserve the WZ gauge in the deformed theory, the gauge transformation has also correction in the form of power series in C .

There exist some interesting cases where the deformation structure becomes simple. One is the limit to the $\mathcal{N} = 1/2$ superspace[5], where the action should reduce to $\mathcal{N} = 1/2$ super Yang-Mills theory with adjoint matter. Another interesting case is the singlet deformation[10, 11], where the deformation parameters belongs to the singlet representation of the R -symmetry group $SU(2)_R$. In this paper, we will study $\mathcal{N} = 2$ supersymmetric $U(1)$ gauge theory in the harmonic superspace with singlet deformation. In this case, the gauge and supersymmetry transformations get correction linear in the deformation parameter. Therefore we can easily perform the field redefinition such that the component fields transform canonically under the gauge transformation. In the case of $\mathcal{N} = 1/2$ super Yang-Mills theory, such field redefinition is also possible[5]. But in this case the component fields do not transform canonically under the deformed supersymmetry transformation. In the singlet case, we will show that there is a field redefinition such that the redefined fields also transform canonically under the deformed supersymmetry. We will construct a deformed Lagrangian which is invariant under both the gauge and supersymmetry transformations. We find that the deformed Lagrangian is characterized by a single function of an anti-holomorphic scalar field.

We begin with reviewing the non(anti)commutative deformation of $\mathcal{N} = 2$ harmonic

superspace [10, 11, 12]. The $\mathcal{N} = 2$ harmonic superspace [16] has coordinates $(x^\mu, \theta_i^\alpha, \bar{\theta}^{\dot{\alpha}i}, u^{\pm i})$, where $\mu = 0, 1, 2, 3$ are spacetime indices, $\alpha, \dot{\alpha} = 1, 2$ spinor indices and $i = 1, 2$ $SU(2)_R$ indices. We will consider the Euclidean signature of spacetime. For lowering and raising spinor indices, we use an antisymmetric tensor $\varepsilon_{\alpha\beta}$ with $\varepsilon^{12} = -\varepsilon_{12} = 1$, while for $SU(2)_R$ indices, we use ϵ_{ij} with $\epsilon^{12} = -\epsilon_{12} = -1$. The coordinates $(x^\mu, \theta_i^\alpha, \bar{\theta}^{\dot{\alpha}i})$ are those of $\mathcal{N} = 2$ rigid superspace. The bosonic variables $u^{\pm i}$, called the harmonic variables, form an $SU(2)$ matrix satisfying $u^{+i}u_i^- = 1$ and $\overline{u^{+i}} = u_i^-$. The harmonic variables are necessary for the off-shell formulation of supersymmetric field theories with extended supersymmetry as developed in [16]. The supersymmetry generators $Q_\alpha^i, \bar{Q}_{\dot{\alpha}i}$ and the supercovariant derivatives $D_\alpha^i, \bar{D}_{\dot{\alpha}i}$ are defined by

$$\begin{aligned} Q_\alpha^i &= \frac{\partial}{\partial \theta_i^\alpha} - i(\sigma^\mu)_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}i} \frac{\partial}{\partial x^\mu}, & \bar{Q}_{\dot{\alpha}i} &= -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}i}} + i\theta_i^\alpha (\sigma^\mu)_{\alpha\dot{\alpha}} \frac{\partial}{\partial x^\mu} \\ D_\alpha^i &= \frac{\partial}{\partial \theta_i^\alpha} + i(\sigma^\mu)_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}i} \frac{\partial}{\partial x^\mu}, & \bar{D}_{\dot{\alpha}i} &= -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}i}} - i\theta_i^\alpha (\sigma^\mu)_{\alpha\dot{\alpha}} \frac{\partial}{\partial x^\mu}. \end{aligned} \quad (1)$$

In the harmonic superspace, we use the supercovariant derivatives

$$D_\alpha^\pm = u_i^\pm D_\alpha^i, \quad \bar{D}_{\dot{\alpha}}^\pm = u_i^\pm \bar{D}_{\dot{\alpha}i}, \quad (2)$$

which are $U(1)$ -projected by using u_i^\pm . For the off-shell formulation of field theories, the basic ingredient is the analytic superfield Φ satisfying $D_\alpha^+ \Phi = \bar{D}_{\dot{\alpha}}^+ \Phi = 0$. The solution of these constraints can be conveniently written of the form $\Phi = \Phi(x_A^\mu, \theta^+, \bar{\theta}^+, u)$ by introducing analytic coordinates

$$x_A^\mu = x^\mu - i(\theta^i \sigma^\mu \bar{\theta}^j + \theta^j \sigma^\mu \bar{\theta}^i) u_i^+ u_j^- = x^\mu - i(\theta^+ \sigma^\mu \bar{\theta}^- + \theta^- \sigma^\mu \bar{\theta}^+), \quad (3)$$

$$\theta_\alpha^\pm = u_i^\pm \theta_\alpha^i, \quad \bar{\theta}_{\dot{\alpha}}^\pm = u_i^\pm \bar{\theta}_{\dot{\alpha}}^i. \quad (4)$$

We now introduce the nonanticommutativity in the $\mathcal{N} = 2$ harmonic superspace by using the $*$ -product:

$$\{\theta_i^\alpha, \theta_j^\beta\}_* = C_{ij}^{\alpha\beta}, \quad (5)$$

with some constants $C_{ij}^{\alpha\beta}$. We assume that the chiral coordinates $x_L^\mu \equiv x^\mu + i\theta_i \sigma^\mu \bar{\theta}^i$ and $\bar{\theta}_{\dot{\alpha}i}$ (anti-)commute with other coordinates

$$[x_L^\mu, x_L^\nu]_* = [x_L^\mu, \theta_i^\alpha]_* = [x_L^\mu, \bar{\theta}^{\dot{\alpha}i}]_* = 0, \quad \{\bar{\theta}^{\dot{\alpha}i}, \bar{\theta}^{\dot{\beta}j}\}_* = \{\bar{\theta}^{\dot{\alpha}i}, \theta_j^\alpha\}_* = 0. \quad (6)$$

Here the $*$ -product realizing this non(anti)commutativity is defined by

$$f * g(\theta) = f(\theta) \exp(P)g(\theta), \quad P = -\frac{1}{2}\overleftarrow{Q}_\alpha^i C_{ij}^{\alpha\beta} \overrightarrow{Q}_\beta^j. \quad (7)$$

The Poisson structure P commutes with the supercovariant derivatives. This deformation preserves chirality. The constants $C_{ij}^{\alpha\beta}$ is a symmetric property $C_{ij}^{\alpha\beta} = C_{ji}^{\beta\alpha}$ and can be decomposed of the form:

$$C_{ij}^{\alpha\beta} = C_{(ij)}^{\alpha\beta} + \frac{1}{4}\epsilon_{ij}\epsilon^{\alpha\beta}C_s. \quad (8)$$

Here the first term $C_{(ij)}^{\alpha\beta}$ is symmetric with respect to i and j (and also α and β). The second term is antisymmetric and is called the singlet deformation introduced in [10, 11], In [11] the deformation with the Poisson structure $P = -\frac{1}{8}\epsilon^{\alpha\beta}\epsilon_{ij}C_s\overleftarrow{D}_\alpha^i\overrightarrow{D}_\beta^j$ has been studied. In this paper we will consider the $*$ -product (7) with the singlet deformation parameter[10]:

$$P = -\frac{1}{8}\epsilon^{\alpha\beta}\epsilon_{ij}C_s\overleftarrow{Q}_\alpha^i\overrightarrow{Q}_\beta^j. \quad (9)$$

The action of $\mathcal{N} = 2$ supersymmetric $U(1)$ gauge theory in this non(anti)commutative harmonic superspace is written in terms of an analytic superfield V^{++} [17]:

$$S = \frac{1}{2} \sum_{n=2}^{\infty} \frac{(-i)^n}{n} \int d^4x d^8\theta du_1 \cdots du_n \frac{V^{++}(\zeta_1, u_1) * \cdots * V^{++}(\zeta_n, u_n)}{(u_1^+ u_2^+) \cdots (u_n^+ u_1^+)} \quad (10)$$

where $\zeta_i = (x_A, \theta_i^+, \bar{\theta}_i^+)$ and $d^8\theta = d^4\theta^+ d^4\theta^-$ with $d^4\theta^\pm = d^2\theta^\pm d^2\bar{\theta}^\pm$. The harmonic integral $\int du$ is defined as in [16]. The action (10) is invariant under the gauge transformation

$$\delta_\Lambda^* V^{++} = -D^{++}\Lambda + i[\Lambda, V^{++}]_*, \quad (11)$$

where the gauge parameter $\Lambda(\zeta, u)$ is also analytic. D^{++} denotes the harmonic derivative $D^{++} = u^{+i} \frac{\partial}{\partial u^{-i}} - 2i\theta^+ \sigma^\mu \bar{\theta}^+ \frac{\partial}{\partial x_A^\mu} + \theta^{+\alpha} \frac{\partial}{\partial \theta^{-\alpha}} + \bar{\theta}^{+\dot{\alpha}} \frac{\partial}{\partial \bar{\theta}^{-\dot{\alpha}}}$. When an analytic superfield is expanded in the Grassmann coordinates, each component field has a harmonic expansion with respect to u_i^\pm . The analytic superfield V^{++} therefore contains infinitely many auxiliary fields. Since the gauge parameter Λ also includes infinitely many fields, one can remove unnecessary auxiliary fields as in the commutative case. We then arrive at the Wess-Zumino(WZ) gauge:

$$\begin{aligned} V_{WZ}^{++}(\zeta, u) = & -i\sqrt{2}(\theta^+)^2 \bar{\phi}(x_A) + i\sqrt{2}(\bar{\theta}^+)^2 \phi(x_A) - 2i\theta^+ \sigma^\mu \bar{\theta}^+ A_\mu(x_A) \\ & + 4(\bar{\theta}^+)^2 \theta^+ \psi^i(x_A) u_i^- - 4(\theta^+)^2 \bar{\theta}^+ \bar{\psi}^i(x_A) u_i^- + 3(\theta^+)^2 (\bar{\theta}^+)^2 D^{ij}(x_A) u_i^- u_j^-. \end{aligned} \quad (12)$$

In [12], we have computed the $O(C)$ action (10) in the WZ gauge explicitly. In the singlet case, the order $O(C_s)$ Lagrangian reads $\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)}$, where

$$\begin{aligned}\mathcal{L}^{(0)} &= -\frac{1}{4}F_{\mu\nu}(F^{\mu\nu} + \tilde{F}^{\mu\nu}) - i\psi^i\sigma^\mu\partial_\mu\bar{\psi}_i - \partial^\mu\phi\partial_\mu\bar{\phi} + \frac{1}{4}D_{ij}D^{ij}, \\ \mathcal{L}^{(1)} &= \frac{1}{\sqrt{2}}C_s A_\nu\partial_\mu\bar{\phi}(F^{\mu\nu} + \tilde{F}^{\mu\nu}) + \frac{i}{\sqrt{2}}C_s\bar{\phi}(\psi^k\sigma^\nu\partial_\nu\bar{\psi}_k) + \frac{i}{\sqrt{2}}C_s(\psi^k\sigma^\nu\bar{\psi}_k)\partial_\nu\bar{\phi} \\ &\quad - \frac{i}{2}C_s\varepsilon^{\alpha\beta}A_\mu(\sigma^\mu\bar{\psi}^k)_\alpha(\sigma^\nu\partial_\nu\bar{\psi}_k)_\beta + \frac{i}{2}C_s\bar{\psi}^i\bar{\psi}^jD_{ij} + \frac{\sqrt{2}}{4}C_sA_\mu A^\mu\partial^2\bar{\phi} - \frac{\sqrt{2}}{4}C_s\bar{\phi}D^{ij}D_{ij},\end{aligned}\tag{13}$$

where $\tilde{F}_{\mu\nu} \equiv \frac{i}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$. Note that $\mathcal{L}^{(0)}$ is the undeformed Lagrangian.

In the commutative case, the gauge transformation with the gauge parameter $\lambda(x_A)$ preserves the WZ gauge. But for generic $C_{ij}^{\alpha\beta}$, $\lambda(x_A)$ does not. In [12], we have constructed the gauge parameter $\Lambda(\zeta, u)$ which preserves the WZ gauge, which is an infinite power series in the deformation parameter C . We will see now the deformed gauge transformation in the singlet case more explicitly. For later convenience, we begin with the deformed gauge transformation (11) of V_{WZ}^{++} with the most general analytic gauge parameter $\Lambda(\zeta, u)$:

$$\begin{aligned}\Lambda(\zeta, u) &= \lambda^{(0,0)}(x_A, u) + \bar{\theta}_\alpha^+\lambda^{(0,1)\dot{\alpha}}(x_A, u) + \theta^{+\alpha}\lambda_\alpha^{(1,0)}(x_A, u) + (\bar{\theta}^+)^2\lambda^{(0,2)}(x_A, u) \\ &\quad + (\theta^+)^2\lambda^{(2,0)}(x_A, u) + \theta^+\sigma^\mu\bar{\theta}^+\lambda_\mu^{(1,1)}(x_A, u) + (\bar{\theta}^+)^2\theta^{+\alpha}\lambda_\alpha^{(1,2)}(x_A, u) \\ &\quad + (\theta^+)^2\bar{\theta}_\alpha^+\lambda^{(2,1)\dot{\alpha}}(x_A, u) + (\theta^+)^2(\bar{\theta}^+)^2\lambda^{(2,2)}(x_A, u).\end{aligned}\tag{14}$$

Here we have denoted the $(\theta^+)^n(\bar{\theta}^+)^m$ -component as $\lambda^{(n,m)}(x_A, u)$. In the case of the singlet deformation, the gauge variation of V_{WZ}^{++} corresponding to this general gauge parameter is calculated as

$$\begin{aligned}\delta_\Lambda^* V_{WZ}^{++} &= -\partial^{++}\lambda^{(0,0)} + \bar{\theta}_\alpha^+(-\partial^{++}\lambda^{(0,1)\dot{\alpha}}) + \theta^{+\alpha}(-\partial^{++}\lambda_\alpha^{(1,0)}) \\ &\quad + (\bar{\theta}^+)^2(iC_s\partial_\mu\lambda^{(0,0)}A^\mu - \partial^{++}\lambda^{(0,2)}) + (\theta^+)^2(-\partial^{++}\lambda^{(2,0)}) \\ &\quad + \theta^+\sigma^\mu\bar{\theta}^+(2i\partial_\mu\lambda^{(0,0)} + \sqrt{2}iC_s\partial_\mu\lambda^{(0,0)}\bar{\phi} - \partial^{++}\lambda_\mu^{(1,1)}) \\ &\quad + (\bar{\theta}^+)^2\theta^{+\alpha}\left(-2C_s\partial_\mu\lambda^{(0,0)}(\sigma^\mu\bar{\psi}^i)_\alpha u_i^- - i(\sigma^\nu\partial_\nu\lambda^{(0,1)})_\alpha - \frac{i}{\sqrt{2}}C_s(\sigma^\nu\partial_\nu\lambda^{(0,1)})_\alpha\bar{\phi}\right. \\ &\quad \left.+ \frac{i}{2}C_s\lambda^{(1,0)\beta}(\sigma^\mu\bar{\sigma}^\nu\varepsilon)_{\beta\alpha}\partial_\mu A_\nu + iC_s\partial_\mu\lambda_\alpha^{(1,0)}A_\mu - \partial^{++}\lambda_\alpha^{(1,2)}\right) \\ &\quad + (\theta^+)^2\bar{\theta}_\alpha^+\left(i\partial_\mu\lambda^{(1,0)\beta}\sigma_{\beta\dot{\beta}}^\mu\varepsilon^{\dot{\beta}\dot{\alpha}} + \frac{i}{\sqrt{2}}C_s\partial_\nu\left\{\lambda^{(1,0)\alpha}\sigma_{\alpha\dot{\beta}}^\nu\varepsilon^{\dot{\beta}\dot{\alpha}}\bar{\phi}\right\} - \partial^{++}\lambda^{(2,1)\dot{\alpha}}\right)\end{aligned}$$

$$\begin{aligned}
& + (\theta^+)^2 (\bar{\theta}^+)^2 \left(C_s \partial_\nu \left\{ \lambda^{(1,0)\alpha} (\sigma^\nu \bar{\psi}^-)_\alpha \right\} - i \partial^\mu \lambda_\mu^{(1,1)} \right. \\
& \quad \left. - \frac{i}{\sqrt{2}} C_s \partial^\mu (\lambda_\mu^{(1,1)} \bar{\phi}) + i C_s \partial^\mu (\lambda^{(2,0)} A_\mu) - \partial^{++} \lambda^{(2,2)} \right), \tag{15}
\end{aligned}$$

where $\partial^{++} \equiv u^{+i} \frac{\partial}{\partial u^{-i}}$. Requiring that the analytic gauge parameter preserves the WZ gauge, we find that the gauge parameter should satisfy $\lambda^{(0,0)}(x_A, u) = \lambda(x_A)$ and the other components are zero. Namely, the analytic gauge parameter retaining the WZ gauge is of the same form as in the commutative case:

$$\Lambda(\zeta, u) = \lambda(x_A). \tag{16}$$

Then we immediately find the deformed gauge transformation laws for the component fields in the case of the singlet deformation:

$$\begin{aligned}
\delta_\Lambda^* A_\mu &= - \left(1 + \frac{1}{\sqrt{2}} C_s \bar{\phi} \right) \partial_\mu \lambda, \quad \delta_\Lambda^* \phi = \frac{1}{\sqrt{2}} C_s A_\mu \partial^\mu \lambda, \quad \delta_\Lambda^* \psi_\alpha^i = - \frac{1}{2} C_s \partial_\mu \lambda (\sigma^\mu \bar{\psi}^i)_\alpha, \\
\delta_\Lambda^* \bar{\phi} &= \delta_\Lambda^* \bar{\psi}_\alpha^i = \delta_\Lambda^* D^{ij} = 0. \tag{17}
\end{aligned}$$

Note that there is no higher order correction in C_s .

We will determine the supersymmetry transformation δ_ξ generated by the supersymmetry generators Q_α^i . First we consider the action of Q_α^i on the gauge superfield:

$$\tilde{\delta}_\xi V_{WZ}^{++} \equiv \xi_i^\alpha Q_\alpha^i V_{WZ}^{++}. \tag{18}$$

In the analytic basis,

$$\xi_i^\alpha Q_\alpha^i = -\xi^{+\alpha} Q_\alpha^- + \xi^{-\alpha} Q_\alpha^+, \tag{19}$$

where $\xi_\alpha^\pm \equiv \xi_\alpha^i u_i^\pm$ and

$$Q_\alpha^+ = \frac{\partial}{\partial \theta^{-\alpha}} - 2i \sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{+\dot{\alpha}} \frac{\partial}{\partial x_A^\mu}, \quad Q_\alpha^- = -\frac{\partial}{\partial \theta^{+\alpha}}. \tag{20}$$

The variation is not affected by the nonanticommutativity, so that we have

$$\begin{aligned}
\tilde{\delta}_\xi V_{WZ}^{++} &= \bar{\theta}_\alpha^+ \left(2i (\xi^+ \sigma^\mu)_{\dot{\beta}} \varepsilon^{\beta\dot{\alpha}} A_\mu \right) + \theta^{+\alpha} \left(-2\sqrt{2} i \xi_\alpha^+ \bar{\phi} \right) + (\bar{\theta}^+)^2 \left(4 \xi^+ \psi^i u_i^- \right) \\
&+ \theta^+ \sigma^\mu \bar{\theta}^+ \left(4 \xi^+ \sigma_\mu \bar{\psi}^i u_i^- \right) + (\bar{\theta}^+)^2 \theta^{+\alpha} \left(-2 (\sigma^\mu \bar{\sigma}^\nu \xi^-)_\alpha \partial_\nu A_\mu + 6 \xi_\alpha^+ D^{ij} u_i^- u_j^- \right) \\
&+ (\theta^+)^2 \bar{\theta}_\alpha^+ \left(2\sqrt{2} (\xi^- \sigma^\mu)_{\dot{\beta}} \varepsilon^{\beta\dot{\alpha}} \partial_\mu \bar{\phi} \right) + (\theta^+)^2 (\bar{\theta}^+)^2 \left(-4 i \xi^- \sigma^\mu \partial_\mu \bar{\psi}^i u_i^- \right). \tag{21}
\end{aligned}$$

This is out of the WZ gauge, so in order to retain the WZ gauge it should be associated with an appropriate deformed gauge transformation δ_Λ^* , as in the commutative case. From the result for the deformed gauge variation with the most general gauge parameter, (15), we can find that at least $\lambda^{(0,0)}$ and $\lambda^{(2,0)}$ should be zero. With the use of such an analytic gauge parameter, the equations to determine the deformed supersymmetric transformation of the component fields will be found from the equation

$$\delta_\xi V_{WZ}^{++} = \tilde{\delta}_\xi V_{WZ}^{++} + \delta_\Lambda^* V_{WZ}^{++}. \quad (22)$$

From those equations, we can find the appropriate gauge parameter is the one as same as in the $C_s = 0$ case and determine the deformed supersymmetry transformations as

$$\begin{aligned} \delta_\xi A_\mu &= i\xi^i \sigma_\mu \bar{\psi}_i, \quad \delta_\xi \phi = -\sqrt{2}i\xi^i \psi_i, \quad \delta_\xi \bar{\phi} = 0, \\ \delta_\xi \psi_\alpha^i &= \left(1 + \frac{1}{\sqrt{2}}C_s \bar{\phi}\right) (\sigma^{\mu\nu} \xi^i)_\alpha F_{\mu\nu} - D^{ij} \xi_{\alpha j} + \frac{1}{\sqrt{2}}C_s \xi_\alpha^i \partial_\mu \bar{\phi} A^\mu, \\ \delta_\xi \bar{\psi}^{\dot{\alpha}i} &= -\sqrt{2}(\bar{\sigma}^\mu \xi^i)^{\dot{\alpha}} \left(1 + \frac{1}{\sqrt{2}}C_s \bar{\phi}\right) \partial_\mu \bar{\phi}, \\ \delta_\xi D^{kl} &= -i\xi^k \sigma^\mu \partial_\mu \left\{ \bar{\psi}^l \left(1 + \frac{1}{\sqrt{2}}C_s \bar{\phi}\right) \right\} - i\xi^l \sigma^\mu \partial_\mu \left\{ \bar{\psi}^k \left(1 + \frac{1}{\sqrt{2}}C_s \bar{\phi}\right) \right\}, \end{aligned} \quad (23)$$

where $\sigma^{\mu\nu} \equiv \frac{1}{4}(\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu)$.

We have seen that the deformed gauge and supersymmetry transformations are exact at the order $O(C_s)$. One may consider the component action which is invariant under these transformations. The variation of $\mathcal{L}^{(1)}$ for the deformed gauge and supersymmetry transformations produces new terms of order $O(C_s^2)$, which should be cancelled by the variation of the $O(C_s^2)$ Lagrangian $\mathcal{L}^{(2)}$. But it turns out that these deformed transformations change only the $\bar{\phi}$ dependence of interaction terms among gauge fields and fermions. Since the spacetime coordinates have noncommutativity with nilpotent parameters, we expect that the Lagrangian do not include the higher derivative terms. Thus we assume the Lagrangian takes the form

$$\begin{aligned} \mathcal{L} = & f_1(\bar{\phi}) F_{\mu\nu} (F^{\mu\nu} + \tilde{F}^{\mu\nu}) + f_2(\bar{\phi}) \psi^i \sigma^\mu \partial_\mu \bar{\psi}_i + f_3(\bar{\phi}) \phi + f_4(\bar{\phi}) D_{ij} D^{ij} \\ & + f_5(\bar{\phi}) A_\mu (\bar{\psi}^i \bar{\sigma}^\mu \sigma^\nu \partial_\nu \bar{\psi}_i) + f_6(\bar{\phi}) A_\mu \partial_\nu \bar{\phi} (F^{\mu\nu} + \tilde{F}^{\mu\nu}) + f_7(\bar{\phi}) A^\mu A_\mu + f_8^{\mu\nu}(\bar{\phi}) A_\mu A_\nu \\ & + f_9\mu(\bar{\phi}) \psi^i \sigma^\mu \bar{\psi}_i + f_{10\nu}(\bar{\phi}) A_\mu \bar{\psi}^i \bar{\sigma}^\mu \sigma^\nu \bar{\psi}_i + f_{11}(\bar{\phi}) D_{ij} \bar{\psi}^i \bar{\psi}^j + f_{12}(\bar{\phi}) (\bar{\psi}^i \bar{\psi}^j) (\bar{\psi}_i \bar{\psi}_j). \end{aligned} \quad (24)$$

Here f_i are functions of $\bar{\phi}$ and its derivatives. Invariance of the Lagrangian \mathcal{L} under the deformed gauge transformation (17) imposes the constraints on the functions f_i 's. Similarly invariance under the deformed supersymmetry transformation (23) leads to the further constraints on f_i 's. From the set of those constraints, f_i 's are solved by the function f_2 . Therefore the Lagrangian becomes

$$\begin{aligned}
\mathcal{L} = & i f_2(\bar{\phi}) \left\{ -\frac{1}{4} \left(1 + \frac{1}{\sqrt{2}} C_s \bar{\phi} \right) F_{\mu\nu} (F^{\mu\nu} + \tilde{F}^{\mu\nu}) - i \psi^i \sigma^\mu \partial_\mu \bar{\psi}_i + \left(1 + \frac{1}{\sqrt{2}} C_s \bar{\phi} \right) \partial^2 \bar{\phi} \phi \right. \\
& + \frac{1}{4} \frac{1}{1 + \frac{1}{\sqrt{2}} C_s \bar{\phi}} D_{ij} D^{ij} - \frac{\frac{i C_s}{2}}{1 + \frac{1}{\sqrt{2}} C_s \bar{\phi}} A_\mu (\bar{\psi}^k \bar{\sigma}^\mu \sigma^\nu \partial_\nu \bar{\psi}_k) - \frac{1}{\sqrt{2}} C_s A_\mu \partial_\nu \phi (F^{\mu\nu} + \tilde{F}^{\mu\nu}) \\
& + \frac{C_s}{2\sqrt{2}} \left\{ \partial^\mu \partial_\mu \bar{\phi} - \frac{\frac{C_s}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}} C_s \bar{\phi}} \partial^\mu \bar{\phi} \partial_\mu \bar{\phi} \right\} A^\mu A_\mu + \frac{1}{4} \frac{C_s^2}{1 + \frac{1}{\sqrt{2}} C_s \bar{\phi}} \partial_\mu \bar{\phi} \partial_\nu \bar{\phi} A^\mu A^\nu \\
& + \frac{i \frac{C_s}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}} C_s \bar{\phi}} \partial^\mu \bar{\phi} \psi^k \sigma_\mu \bar{\psi}_k + \frac{i \frac{C_s^2}{2\sqrt{2}}}{\left(1 + \frac{1}{\sqrt{2}} C_s \bar{\phi} \right)^2} \partial_\nu \bar{\phi} A_\mu \bar{\psi}^k \bar{\sigma}^\mu \sigma^\nu \bar{\psi}_k \\
& \left. + \frac{i}{2} \frac{C_s}{\left(1 + \frac{1}{\sqrt{2}} C_s \bar{\phi} \right)^2} D_{ij} \bar{\psi}^i \bar{\psi}^j - \frac{1}{4} \frac{C_s^2}{\left(1 + \frac{1}{\sqrt{2}} C_s \bar{\phi} \right)^3} (\bar{\psi}^i \bar{\psi}^j) (\bar{\psi}_i \bar{\psi}_j) \right\}. \tag{25}
\end{aligned}$$

At the order $O(C_s)$, the Lagrangian reduced to the result (13) due to $f_2 = -i \left(1 - \frac{1}{\sqrt{2}} C_s \bar{\phi} \right) + O(C_s^2)$.

Now we consider the field redefinition. In the singlet deformation case, the $O(C_s)$ gauge transformation (17) is exact. We can redefine the component fields such that these transform canonically under the deformed gauge transformation. Let us introduce \hat{A}_μ , $\hat{\phi}$ and $\hat{\psi}^i$ by

$$\hat{A}_\mu = F(\bar{\phi}) A_\mu, \quad \hat{\phi} = \phi + G(\bar{\phi}) A_\mu A^\mu, \quad \hat{\psi}_\alpha^i = \psi_\alpha^i + H(\bar{\phi}) A^\mu (\sigma^\mu \bar{\psi})_\alpha, \tag{26}$$

where $F(\bar{\phi})$, $G(\bar{\phi})$ and $H(\bar{\phi})$ are functions of $\bar{\phi}$. If we require that these fields transform canonically, *i.e.* $\delta_\lambda \hat{A}_\mu = -\partial_\mu \lambda$, and $\delta_\lambda \hat{\phi} = \delta_\lambda \hat{\psi}^i = 0$, then the functions F , G and H are determined as

$$F(\bar{\phi}) = \frac{1}{1 + \frac{1}{\sqrt{2}} C_s \bar{\phi}}, \quad G(\bar{\phi}) = \frac{\frac{1}{2\sqrt{2}} C_s}{1 + \frac{1}{\sqrt{2}} C_s \bar{\phi}}, \quad H(\bar{\phi}) = -\frac{\frac{1}{2} C_s}{1 + \frac{1}{\sqrt{2}} C_s \bar{\phi}}. \tag{27}$$

It is easy to see that the Lagrangian (25) after the field redefinitions (26) also coincides with the $O(C_s)$ result in [12].

The $\mathcal{N} = 2$ vector multiplet $(D^{ij}, \hat{A}_\mu, \hat{\psi}^i, \bar{\psi}_i, \hat{\phi}, \bar{\phi})$, however, does not transform canonically under the supersymmetry transformation. But, if we instead perform field redefinitions as

$$\begin{aligned} a_\mu &= F(\bar{\phi})A_\mu, \quad \varphi = F(\bar{\phi})^2 \left(\phi + G(\bar{\phi})A_\mu A^\mu \right), \quad \bar{\varphi} = \bar{\phi}, \\ \lambda_\alpha^i &= F(\bar{\phi})^2 \left(\psi_\alpha^i + H(\bar{\phi})A_\mu (\sigma^\mu \bar{\psi}^i)_\alpha \right), \quad \bar{\lambda}^{\dot{\alpha}i} = F(\bar{\phi})\bar{\psi}^{\dot{\alpha}i}, \\ \tilde{D}^{ij} &= F(\bar{\phi})^2 \left(D^{ij} - 2iH(\bar{\phi})\bar{\psi}^i \bar{\psi}^j \right), \end{aligned} \quad (28)$$

we can show that the multiplet $(\tilde{D}^{ij}, a_\mu, \lambda^i, \bar{\lambda}_i, \varphi, \bar{\varphi})$ now transforms canonically under the supersymmetry transformation as well as the gauge transformation:

$$\begin{aligned} \delta_\xi a_\mu &= i\xi^i \sigma_\mu \bar{\lambda}_i, \quad \delta_\xi \varphi = -i\sqrt{2}\xi^i \lambda_i, \quad \delta_\xi \bar{\varphi} = 0, \\ \delta_\xi \lambda_\alpha^i &= (\sigma^{\mu\nu} \xi^i)_\alpha f_{\mu\nu} - \tilde{D}^{ij} \xi_{j\alpha}, \quad \delta_\xi \bar{\lambda}^{\dot{\alpha}i} = -\sqrt{2}(\bar{\sigma}^\mu \xi^i)^{\dot{\alpha}} \partial_\mu \bar{\varphi}, \\ \delta_\xi \tilde{D}^{ij} &= -i \left(\xi^i \sigma^\mu \partial_\mu \bar{\lambda}^j + \xi^j \sigma^\mu \partial_\mu \bar{\lambda}^i \right), \end{aligned} \quad (29)$$

here $f_{\mu\nu} \equiv \partial_\mu a_\nu - \partial_\nu a_\mu$. In terms of these newly defined fields, the Lagrangian becomes

$$\mathcal{L} = if_2(\bar{\varphi}) \left(1 + \frac{1}{\sqrt{2}} C_s \bar{\varphi} \right)^3 \left\{ -\frac{1}{4} f_{\mu\nu} (f^{\mu\nu} + \tilde{f}^{\mu\nu}) - i\lambda^i \sigma^\mu \partial_\mu \bar{\lambda}_i + \varphi \partial^2 \bar{\varphi} + \frac{1}{4} \tilde{D}_{ij} \tilde{D}^{ij} \right\}, \quad (30)$$

which takes a simple form.

In this paper, we have determined the deformed gauge and supersymmetry transformation of component fields of $\mathcal{N} = 2$ supersymmetric $U(1)$ gauge theory in the noncommutative harmonic superspace with the singlet deformation parameter. The Lagrangian which is invariant under these transformations are obtained. In this work, we could not determine the complete Lagrangian due to the function $f_2(\bar{\phi})$, which is necessary for further study. We have studied the field redefinition of component fields, such that these fields transform canonically under the gauge transformation. It is interesting to compare the present result to the Lagrangian in [11], which is based on the different Poisson structure.

It is also interesting to study the deformed supersymmetry in the case of non-singlet deformation. We expect that the action is invariant under the deformed $\mathcal{N} = (1, 0)$ supersymmetry. In this case one may consider the $\mathcal{N} = 1/2$ superspace limit by restricting $C_{ij}^{\alpha\beta}$ to $C^{\alpha\beta} \delta_i^1 \delta_j^1$. The action in this limit is expected to reduce to that in [6], in which

it was claimed that the $\mathcal{N} = 2$ action has only $\mathcal{N} = 1/2$ supersymmetry. In [6], the supersymmetry linearly deformed in C has been examined. But the reduction from the harmonic superspace suggests that the deformed supersymmetry is realized nonlinearly in C . In a subsequent paper [18], we will study the structure of supersymmetry for generic deformation parameters and clarify this point.

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Note added: After this paper was submitted to the e-archive, a new paper has appeared [19] where the undetermined function in the component Lagrangian is completely fixed.

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